The ϕCDM Model : A light or massless scalar field coupling to matter and cold dark matter

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Abstract

The ϕCDM model is one explanation among others that have been proposed to solve the problem of the origin of Dark Energy. We review the main aspects of the CDM model. We examine in details the reasons behind the introduction of the dilaton field. We address how the dilaton field may couple to matter and to Dark Matter. We investigate how this coupling results in a variation of Newtons constant. As this dilaton field has not been observed yet we study the possible symmetry constraints on couplings of the dilaton field with the usual Standard-Model fields. For the allowed couplings, we obtain bounds on the values of the corresponding coupling constants. Some bounds are obtained by deriving the impact of these couplings on the polarisation of quasar light. Other bounds are obtained thanks to the influence of the dilaton field on the Cosmological Microwave Background. All the resulting bounds are compared to observational data.

Conventions

Writing conventions

Metric

We shall use the $(-,+,+,+)$ convention for the special relativity metric: $\boldsymbol{\eta} = \mathrm{diag}(-1,1,1,1)$.

Tensors

- Bold letters will denote tensors: $\mathbf{G}, \mathbf{g}$, ... and they can also have indices.
- If a bold letter represents a given tensor, the same letter non-bold and italic will denote its determinant. For example, $\det \mathbf{g} = g$.
- As we are using tensors, it is convenient to use Einstein summation convention for repeated indices. For example, with two rank-1 tensors:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{A}_\mu \mathbf{B}^\mu = \sum_\mu \mathbf{A}_\mu \mathbf{B}^\mu$$

Operators

- Nabla denotes the covariant derivative and is, as such, a tensor: ∇, ∇_μ...
- The partial derivative (non-covariant) will be denoted: ∂_μ.
- We will also use the dalambertian operator: $\Box = \nabla_\mu \nabla^\mu$.
- The two above derivatives act on every term on its right: $\nabla_\mu \phi\psi = \nabla_\mu(\phi\psi)$, $\nabla_\mu \nabla_\nu \phi = \nabla_\mu(\nabla_\nu \phi)$...
- As every operator does: $\mathrm{Tr}\,\mathbf{AB} = \mathrm{Tr}(\mathbf{AB})$

When a rank-1 tensor is squared it is with the scalar product with respect to the metric:

$$(\nabla\phi)^2 = \nabla_\mu \phi \nabla^\mu \phi = \mathbf{g}_{\mu\nu} \nabla^\nu \phi \nabla^\mu \phi.$$

The trace of a rank-2 tensor is with regard to the metric:

$$\mathrm{Tr}\,\mathbf{A} = \mathbf{g}_{\mu\nu} A^{\mu\nu}.$$

- Ricci scalar curvature: $\mathcal{R}$
- Ricci tensor: **Ric**
- Einstein tensor: **G**

The above conventions are true unless otherwise indicated.

Constants

Physical constants	**Notation**	**Value and unit**
Speed of light	c	1
Newton gravitationnal constant	$\mathscr{G}$	7.5×10^{28} kg/m

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Chapter 1

Introduction

THANKS TO THE OBSERVATION of Type Ia Supernovæ in 1998, it has been experimentally shown that the expansion of the universe is accelerating [10]. Even though expansion of the universe was already known since the observations of Edwin HUBBLE in 1929, this acceleration was unpredicted by the theories in use in the nineties. An explanation for the expansion of the universe was to add in EINSTEIN's equations a negative pressure called the cosmological constant Λ. This modification was already possible in EINSTEIN's theory.

However, since 1990 there already existed an alternative cosmological model with a massless or nearly massless scalar field. This field proposed by T. DAMOUR, G. W. GIBBONS and C. GUNDLACH in [1] has later been suggested to be responsible for most of the energy density of the universe [2]. The energy contribution of the field that could explain the acceleration of expansion has been coined *dark energy* by Michael TURNER in 1998. This idea is to replace NEWTON's gravitation constant with a field ϕ that varies in space and time. The original idea for the variable gravitational constant was developed by BRANS and DICKE in [6] in order to better understand the role of MACH's principle in general relativity. This scalar field was thus originally called the *dilaton* field. It has later been called a *quintessence* field and more recently *dark energy*. Here we will refer to this field as *dilaton* field.

The model proposed by T. DAMOUR *et al.* [1] considers, in addition to regular matter, Cold Dark Matter (CDM). Cold dark matter is represented by matter fields that do not couple to electromagnetic, weak and strong interactions and do not radiate energy. Dark matter has been hypothetised in the first half of the twentieth century, to explain star [11] and galaxy velocities [12] that were not consistent with the observed matter distribution and general relativity.

This is the ϕCDM model. This work is dedicated to the study of this model.

In this report we will first go through some preliminaries about physical and philosophically-based considerations about the principle of relativity and the BRANS and DICKE model. In chapter three we will study the consequences of the coupling of the dilaton field to both regular matter and dark matter. This study is carried out by deriving a set of modified FRIEDMANN equations as well as a field equation for ϕ. At this stage we will appreciate the consequences that such a field and coupling can have on the variation of NEWTON's constant. In chapter four we will look at the possible couplings with Standard Model fields *via* the allowed symmetries for ϕ. We will also investigate the current experimentally-obtained constraints on the coupling of the dilaton field with other fields. Finally, in chapter five we will examine the possible solutions for a massive dilaton field in an exponential potential. We will also analyse the influence that such a field could have had on the Cosmic Microwave Background and compare it to observations.

Chapter 2

Preliminaries

THIS CHAPTER GATHERS several preliminary topics on which the ϕCDM theory was built. First we will state MACH's principle. MACH's principle was of major influence for EINSTEIN's general relativity. The status of this principle is however not totally clear in the theory. Secondly we will introduce the weak and strong equivalence principles of general relativity and discuss their different implications. Finally we will introduce the BRANS and DICKE theory for a massless and massive scalar dilaton field.

MACH principle

The most fundamental assumption that we will have to suppose true in this theory is MACH's principle. This principle states that the inertial forces locally experienced in an accelerated frame can be interpretated as the effects of *distant masses* accelerated relative to this frame. The thought experiment described in [6] is useful to understand this principle. Let us imagine empty space with only a laboratory with its experimenter inside. Assuming that the mass of this system is small enough for the weak (gravitational) field approximation to hold (*i.e.* the metric is almost minkowskian), the experimenter would observe the usual laws of physics with her/his experiments. The experimenter could throw something out, let's say a brick, tangentially through the window to induce a rotation of the laboratory. A gyroscope inside the laboratory will then start to rotate relative to the laboratory. The point of view of MACH is that the mass of the *distant* brick seems to have a more important effect than the rest of the laboratory to determe inertial coordinate frames and the behaviour of the gyroscope.

Inspired by MACH's principle, EINSTEIN built his theory of gravitation keeping in mind that distant masses are the only elements that can be used to define inertial frames. In general relativity the effect of distant masses is gravitation through its description of space-time geometry.

Weak and strong equivalence principles

During the construction of general relativity two assumptions have been successively made. The first one, known as the *weak equivalence principle*, states that the inertial mass and the gravitational mass are proportional. Up to an appropriate change of units these masses can be set to be equal. At the beginning of mechanics NEWTON assumed that this principle was true.

The second assumption is the *strong equivalence principle* and it states that the trajectory of a small body in a gravitational field depends only on its initial position and velocity (both in space-time) and not on its constitution (this is thus stronger than the weak principle as no reference to the mass is made). It also states that the outcome of any experiment made in a free-falling laboratory is independent of the velocity and the position of the laboratory.
Nowadays both principles have been experimentally tested and are thought to hold (up to 10^{-16} for the weak one and 10^{-5} for the strong one). At the time of BRANS and DICKE , the strong principle had not been tested yet.

BRANS-DICKE theory

BRANS and DICKE considered the thought experiment described in section 2.1 and estimated that it apparently described an absolute space rather than a relative space. Their conclusion is that one of the following assertions has to be true:

1. There is an absolute space.
2. Some unknown boundary conditions make the thought experiment non-physical.
3. The thought experiment situation is ill-described by general relativity.

The first assumption is massively against the relativity principle and would be a serious setback to the physics since Galileo GALILEI. BRANS and DICKE 's argument in against the second assertion is that the universe appears to be non-uniform and there is no apparent reason why a laboratory could not be placed arbitrarily. They hence preferred to question the exactness of general relativity.

The aim of BRANS and DICKE was to follow the third assertion and elaborate a better description of the thought experiment. They noticed that for a brick falling into a star of mass m at a distance r we have

$$a = \mathscr{G}\frac{m}{r^2}. \tag{2.1}$$

A dimensional argument in terms of the mass distribution of the visible universe of size R and mass M for the brick gives that

$$a \propto \frac{Rmc^2}{Mr^2}. \tag{2.2}$$

Hence we have the relationship, with $c = 1$

$$\mathscr{G}\frac{M}{R} \sim 1. \tag{2.3}$$

As the universe is known to be expanding and inhomogeneous, the ratio $\frac{M}{R}$ has no reason to be constant. Hence NEWTON's gravitational constant $\mathscr{G}$ should be able to vary with space and time.

If the gravitational "constant" is to vary, it should be a function of a scalar field. The problem is that no scalar field from general relativity is suitable as these fields decrease faster than $\frac{1}{r}$ from a mass source, and a test mass will be more sensitive to local matter than to *distant matter*, as MACH's principle requires. We hence have to introduce a new scalar field ϕ.
In order to have a field that behaves roughly as $\frac{1}{R}$, it BRANS and DICKE assume that ϕ varies as $\mathscr{G}^{-1}$. We can now try to modify the usual action corresponding to EINSTEIN field equation.

$$\mathcal{S} = \int \left(\mathcal{R} + 16\pi\mathscr{G}\mathscr{L}_M\right)\sqrt{-g}d^4x \tag{2.4}$$

where $\mathscr{L}_M$ is the lagrangian density of matter including all non-gravitational fields, $\mathcal{R}$ is the RICCI scalar curvature and g is the determinant of the metric tensor $\mathbf{g}$. We first divide the lagrangian by $\mathscr{G}$, and then we add DICKE's lagrangian contribution for a massless scalar relativistic field with coupling constant ω which is expected to be of the order of unity [6].

$$\mathcal{S} = \int \left(\mathcal{R}\phi - \omega\frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi} + 16\pi\mathscr{L}_M\right)\sqrt{-g}d^4x. \tag{2.5}$$

One can notice that with such an action, the scalar field is not coupled directly to matter and is only coupled to the geometry. This action, when extremised with respect to ϕ yields a field equation with two terms that can be seen as sources.

$$\Box\phi = -\frac{\mathcal{R}}{2\omega} - \frac{\nabla_\mu\phi\nabla^\mu\phi}{2\phi}. \tag{2.6}$$

The action can also be written more explicitly

$$\mathcal{S} = \int \left(\mathcal{R}\phi - \omega\frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi}\right)\sqrt{-g}d^4x + 16\pi\mathcal{S}_M\left[\psi, \mathbf{g}_{\mu\nu}\right], \tag{2.7}$$

where ψ is all the matter and other Standard Model fields.

Generalising BRANS-DICKE theory to a massive field

For a massive scalar field, the BRANS and DICKE action should be modified in the following way

$$\mathcal{S} = \int \left(\mathcal{R}\phi - \omega\frac{\nabla_\mu\phi\nabla^\mu\phi}{\phi} - V(\phi)\right)\sqrt{-g}d^4x + 16\pi\mathcal{S}_M\left[\psi, \mathbf{g}_{\mu\nu}\right], \tag{2.8}$$

where V is a potential energy.

Transforming the action

To have a more practical expression of the action we perform a Weyl conformal transform with the following change of notation

$$\begin{aligned} \tilde{\phi} &= \tfrac{\phi}{16\pi} \\ \mathbf{g}_{\mu\nu} &= 16\pi\mathcal{G}\tilde{\phi}\tilde{\mathbf{g}}_{\mu\nu} = 2\kappa^2\tilde{\phi}\tilde{\mathbf{g}}_{\mu\nu} \\ \sigma &= -\sqrt{\omega + \tfrac{3}{2}}\ln(2\kappa^2\tilde{\phi}) \end{aligned} \tag{2.9}$$

where $\mathcal{G}$ is a constant of the same dimensions as $\mathscr{G}$ and $\tilde{\mathbf{g}}$ is the previously used metric. We get the following action

$$\mathcal{S} = \int \left(\frac{\mathcal{R}}{2\kappa^2} - \frac{\nabla_\mu\sigma\nabla^\mu\sigma}{2\kappa^2} \right) \sqrt{-g} d^4x + \mathcal{S}_M \left[\psi, e^{2\beta\sigma}\mathbf{g}_{\mu\nu} \right], \tag{2.10}$$

with $\beta = \sqrt{2(\omega + \frac{3}{2})}^{-1}$.
We see that with such an action, the coupling is only *metric*, *i.e.* the dilaton field is only present via a modification of the metric in the regular matter action.
Weyl conformal transforms are transformations of the metric tensor of the following form

$$\tilde{\mathbf{g}}_{\mu\nu} = f(\phi)\mathbf{g}_{\mu\nu}.$$

Every quantity that depends on the metric will thus be affected, for example the Ricci scalar and tensor curvature. These transformations are used to go from the Jordan to the Einstein frames, which are two equivalent descriptions of general relativity.

Chapter 3

Coupling dilatons to matter

Let us study the effective coupling of the dilaton field to matter and dark matter. This interaction is not direct. It is rather due to the coupling between the curvature and the dilaton field on the one hand and on the other hand both kinds of matter field. We will find dynamical equation for both the new space-time and the dilaton field. We will then deduce the implications on Newton's constant.

Coupling to dark matter

Assuming that the other laws of physics hold unchanged, one needs to introduce dark matter to explain inconsistencies between the speed of celestial bodies and the observed matter distribution that have been reported in the nineteen twenties and thirties [11, 12]. In this context the field introduced by Brans and Dicke is likely to couple with both matter and dark matter, but there is no reason for the coupling to be the same for both kinds of matter. We thus have an action as the following

$$\mathcal{S} = \int \left(\mathcal{R}\phi - \omega \frac{\nabla_\mu \phi \nabla^\mu \phi}{\phi} \right) \sqrt{-g} d^4x + 16\pi \mathcal{S}_M \left[\psi_M, \mathbf{g}_{\mu\nu} \right] + 16\pi \mathcal{S}_{DM} \left[\psi_{DM}, \mathbf{g}_{\mu\nu} \right]. \tag{3.1}$$

Performing the Weyl transform like in the previous chapter, we get

$$\mathcal{S} = \int \left(\frac{\mathcal{R}}{2\kappa^2} - \frac{\nabla_\mu \sigma \nabla^\mu \sigma}{2\kappa^2} \right) \sqrt{g} d^4x + \mathcal{S}_{DM} \left[\psi, e^{2\beta_{DM}\sigma} \mathbf{g}_{\mu\nu} \right] + \mathcal{S}_M \left[\psi, e^{2\beta_M \sigma} \mathbf{g}_{\mu\nu} \right]. \tag{3.2}$$

Here we assume that the coupling constants with dark matter and regular matter are different, hence the different coupling constants of the field with matter (β_M) and with dark matter (β_{DM}).

The modified Einstein equation

Extremising the above action for a variation of the metric $\mathbf{g} \mapsto \mathbf{g} + \delta\mathbf{g}$ gives

$$\mathbf{G}_{\mu\nu} = \nabla_\mu \sigma \nabla_\nu \sigma - \frac{1}{2} \mathbf{g}_{\mu\nu} (\nabla\sigma)^2 + \kappa^2 (\mathbf{T}_{M\mu\nu} + \mathbf{T}_{DM\mu\nu}). \tag{3.3}$$

For a variation of the σ field, we get the following field equation

$$\Box\sigma = -\kappa^2 \mathbf{g}^{\mu\nu} (\beta_M \mathbf{T}_{M\mu\nu} + \beta_{DM} \mathbf{T}_{DM\mu\nu}), \tag{3.4}$$

where $\mathbf{G}$ is the Einstein tensor and the energy-momentum tensors of matter and dark matter are defined as follows

$$\mathbf{T}_M^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta \mathcal{S}_M \left[\psi, e^{2\beta_M \sigma} \mathbf{g}_{\mu\nu} \right]}{\delta \mathbf{g}_{\mu\nu}} \qquad \text{and} \qquad \mathbf{T}_{DM}^{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta \mathcal{S}_{DM} \left[\psi, e^{2\beta_{DM} \sigma} \mathbf{g}_{\mu\nu} \right]}{\delta \mathbf{g}_{\mu\nu}}.$$

From those field equations, we get

$$\nabla^\mu \mathbf{T}_{M\mu\nu} = \beta_M \operatorname{Tr}(\mathbf{T}_M) \nabla_\mu \sigma \qquad \text{and} \qquad \nabla^\mu \mathbf{T}_{DM\mu\nu} = \beta_{DM} \operatorname{Tr}(\mathbf{T}_{DM}) \nabla_\mu \sigma. \tag{3.5}$$

Derivation of Friedmann equations and the field equation

Let us look at the case of a perfect fluid distribution, thus described by the energy-momentum tensors of matter and dark matter

$$\mathbf{T}_M^{\mu\nu} = (\rho_M + P_M) u_M^\mu u_M^\nu + P_M \mathbf{g}^{\mu\nu} \qquad \text{and} \qquad \mathbf{T}_{DM}^{\mu\nu} = (\rho_{DM} + P_{DM}) u_{DM}^\mu u_{DM}^\nu + P_{DM} \mathbf{g}^{\mu\nu} \tag{3.6}$$

where the ρ's are the densities and the P's the pressures of matter and dark matter, we also have $u_M^\mu u_{M\mu} = u_{DM}^\mu u_{DM\mu} = -1$.

One can consider the large-scale structure of the universe as homogeneous ans isotropic we hence use the FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER metric defined as follows,

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right), \tag{3.7}$$

where $a(t)$ is a time dependent scale factor and k can take different values depending on the curvature we choose to model the universe ($k = 1$ for a positive curvature, $k = 0$ for a flat space-time and $k = -1$ for a negative curvature). We thus have the following **covariant** tensors (indices downstairs), in spherical coordinates

$$\mathbf{g} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-kr^2} & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2\sin^2\theta \end{pmatrix} \tag{3.8}$$

$$\mathbf{Ric} = \begin{pmatrix} -3\frac{\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & \frac{a\ddot{a}+2\dot{a}^2+2k}{1-kr^2} & 0 & 0 \\ 0 & 0 & r^2(a\ddot{a}+2\dot{a}^2+2k) & 0 \\ 0 & 0 & 0 & r^2(a\ddot{a}+2\dot{a}^2+2k)\sin^2\theta \end{pmatrix} \tag{3.9}$$

$$\mathcal{R} = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \tag{3.10}$$

And the EINSTEIN tensor thus reads

$$\mathbf{G} = \mathbf{Ric} - \frac{1}{2}\mathcal{R}\mathbf{g} = \begin{pmatrix} 3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) & 0 & 0 & 0 \\ 0 & \frac{2a\ddot{a}+\dot{a}^2+k}{kr^2-1} & 0 & 0 \\ 0 & 0 & -r^2\left(2a\ddot{a}^2+\dot{a}^2+k\right) & 0 \\ 0 & 0 & 0 & -\left(2a\ddot{a}^2+\dot{a}^2+k\right)r^2\sin^2\theta \end{pmatrix} \tag{3.11}$$

and the energy-momentum tensors read (with $u_\mu = (1,0,0,0)$ to respect space-isotropy)

$$\mathbf{T} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1-kr^2}P & 0 & 0 \\ 0 & 0 & Pa^2(t)r^2 & 0 \\ 0 & 0 & 0 & Pa^2(t)r^2\sin^2\theta \end{pmatrix} \tag{3.12}$$

Using equation (3.3) and assuming that $u^\mu_M = u^\mu_{DM}$, we get the following equations. The time coordinate term yields

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa^2(\rho_M + \rho_{DM}) + \frac{1}{2}(\nabla\sigma)^2 + (\partial_t\sigma)^2 \tag{3.13}$$

The radial coordinate term yields

$$\frac{2a\ddot{a}+\dot{a}^2+k}{kr^2-1} = \kappa^2\frac{a^2(t)}{1-kr^2}(P_M + P_{DM}) - \frac{1}{2}(\nabla\sigma)^2\frac{a^2(t)}{1-kr^2} + (\partial_r\sigma)^2 \tag{3.14}$$

and the angular coordinate term yields the two following equations

$$-r^2\left(2a\ddot{a}+\dot{a}^2+k\right) = \kappa^2a^2(t)r^2(P_M+P_{DM}) - \frac{1}{2}(\nabla\sigma)^2a^2(t)r^2 + (\partial_\theta\sigma)^2 \tag{3.15}$$

$$-r^2\left(2a\ddot{a}+\dot{a}^2+k\right)\sin^2\theta = \kappa^2a^2(t)r^2(P_M+P_{DM})\sin^2\theta - \frac{1}{2}(\nabla\sigma)^2a^2(t)r^2\sin^2\theta + (\partial_\varphi\sigma)^2 \tag{3.16}$$

If we also suppose that all physical quantities depend only on time, as the universe stays isotropic and homogeneous, we get that $(\nabla\sigma)^2 = \partial_t\sigma\partial^t\sigma = -(\partial_t\sigma)^2 = -\dot{\sigma}^2$ and thus the above equations becomes:

$$3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa^2(\rho_M + \rho_{DM}) + \frac{1}{2}\dot{\sigma}^2 \tag{3.17}$$

$$-\frac{2a\ddot{a}+\dot{a}^2+k}{a^2} = \kappa^2(P_M + P_{DM}) + \frac{1}{2}\dot{\sigma}^2, \tag{3.18}$$

from the radial component and from both angular coordinates. These are the modified FRIEDMANN equations. Mixing the two FRIEDMANN ($\frac{1}{2}((3.17) + 3\times(3.18))$ equations we get

$$-3\frac{\ddot{a}}{a} = \frac{1}{2}\kappa^2\left((\rho_M + 3P_M) + (\rho_{DM} + 3P_{DM})\right) + \dot{\sigma}^2. \tag{3.19}$$

If we take equation (3.4), we get

$$\Box\sigma = -\kappa^2 \left(\beta_M \left(-\rho_M + 3P_M\right) + \beta_{DM} \left(-\rho_{DM} + 3P_{DM}\right)\right). \tag{3.20}$$

Thus, as $\Box\sigma = \partial_t\partial^t\sigma = -\partial_t^2\sigma = -\ddot{\sigma}$ from homogeneity

$$\ddot{\sigma} = -\kappa^2 \left(\beta_M \left(\rho_M - 3P_M\right) + \beta_{DM} \left(\rho_{DM} - 3P_{DM}\right)\right). \tag{3.21}$$

The authors obtained the following field equation[1]

$$-\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\sigma}\right) = \kappa^2 \left(\beta_M \left(\rho_M - 3P_M\right) + \beta_{DM} \left(\rho_{DM} - 3P_{DM}\right)\right). \tag{3.22}$$

A Dark matter universe

If we now assume that the universe is dynamically dominated by dark matter, we then neglect the regular matter pressure and density.
A usual modelling of a homogeneous and isotropic universe is the *dust universe*. This model is characterised by the fact that it is filled with a perfect fluid – dust – the pressure of which is negligible as the particles of dust only have a collective motion and no relative motion. Similarly as for a dust universe, let us assume that the pressure of dark matter is also zero. With these assumptions we get a new form for equations (3.17), (3.19) and (3.22)

$$\begin{aligned} &3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa^2\rho_{DM} + \frac{1}{2}\dot{\sigma}^2 \\ &-3\frac{\ddot{a}}{a} = \frac{1}{2}\kappa^2\rho_{DM} + \dot{\sigma}^2 \\ &-\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\sigma}\right) = \kappa^2\beta_{DM}\rho_{DM} \end{aligned} \tag{3.23}$$

Let us now proceed to a change of notation in order to simplify the equations. We introduce the Hubble ratio $H = \frac{\dot{a}}{a}$, as well as $y = \dot{\sigma}$ and we change our system of units such that $\kappa^2 = 1$. Doing so in the set of equations (3.23) we obtain the following constrained dynamical system

$$\begin{aligned} &3\left(H^2 + \frac{k}{a^2}\right) = \rho_{DM} + \frac{1}{2}y^2 \\ &-3\frac{\ddot{a}}{a} = \frac{1}{2}\rho_{DM} + y^2 \\ &-\frac{1}{a^3}\frac{d}{dt}\left(a^3 y\right) = \beta_{DM}\rho_{DM} \end{aligned} \tag{3.24}$$

This can be further simplified introducing $F = \frac{k}{a^2H^2}$

$$\begin{aligned} &3H^2\left(1+F\right) = \rho_{DM} + \frac{1}{2}y^2 \\ &-3\left(\dot{H} + H^2\right) = \frac{1}{2}\rho_{DM} + y^2 \\ &\dot{y} = -3Hy - \beta_{DM}\rho_{DM}. \end{aligned} \tag{3.25}$$

Hence, replacing $\rho_{DM} = 3H^2\left(1+F\right) - \frac{1}{2}y^2$ with the first equation we get

$$\begin{aligned} &\dot{y} = -3Hy - \beta_{DM}\left(3H^2\left(1+F\right) - \frac{1}{2}y^2\right) \\ &-3\dot{H} = 3H^2 + \left(\frac{1}{2}\left(3H^2\left(1+F\right) - \frac{1}{2}y^2\right) + y^2\right). \end{aligned}$$

Thus

$$\begin{aligned} &\dot{y} = -3Hy - \beta_{DM}\left(3H^2\left(1+F\right) - \frac{1}{2}y^2\right) \\ &\dot{H} = -\frac{1}{2}H^2\left(3+F\right) - \frac{1}{4}y^2. \end{aligned} \tag{3.26}$$

[1] From my point of view, a term is missing in that equation: $-3\frac{\dot{\sigma}\dot{a}}{a}$ and I can not see any reason to neglect it.

If we now want a third equation, let us look at the derivative of F

$$\begin{aligned}
\dot{F} &= -2k\frac{\ddot{a}}{\dot{a}^3} \\
&= -2k\left(\dot{H}+H^2\right)\frac{a}{\dot{a}^3} \\
&= -2k\left(-\frac{1}{2}H^2\left(1+F\right)-\frac{1}{4}y^2\right)\frac{a}{\dot{a}^3} && \text{with the second equation of (3.26)} \\
&= \left(1+F\right)\frac{k}{a\dot{a}}+\frac{k}{2}y^2\frac{a}{\dot{a}^3} && \text{since} \quad H^2\frac{a}{\dot{a}^3}=\frac{1}{a\dot{a}} \\
&= HF\left(1+F\right)+\frac{k}{2}y^2\frac{a}{\dot{a}^3} && \text{because} \quad HF=\frac{k}{a\dot{a}} \\
&= HF\left(1+F\right)+\frac{F}{2H}y^2
\end{aligned}$$

We thus have the three following coupled differential equations that form a dynamical system

$$\begin{aligned}
\dot{y} &= -3Hy-\beta_{DM}\left(3H^2\left(1+F\right)-\frac{1}{2}y^2\right) \\
\dot{H} &= -\frac{1}{2}H^2\left(3+F\right)-\frac{1}{4}y^2 \\
\dot{F} &= HF\left(1+F\right)+\frac{F}{2H}y^2.
\end{aligned} \tag{3.27}$$

We will investigate the behaviour of a particular solution of this system in the following section.

Consequences on NEWTON's constant

Now that we have derived the dynamical equations that describe the universe with a dilaton field and cold dark matter, it would be nice to obtain a result that we could confront to the real world via experiments.
As the curvature of the universe is, thanks to experimental data and according to the current theories, thought to be exponentially small, we will focus on solutions with $F=0$ (which corresponds to $k=0$). With a closer look at the (3.27) equations we notice that $F=0$ is an invariant plane in phase space.
As explained in [9], the results written in the EINSTEIN frame are not directly comparable to observation. We thus have to go to the JORDAN frame (from now on, the variables with a tilde are the JORDAN frame ones)[2].
We have

$$d\tilde{s}^2 = e^{2\beta_M\sigma}ds^2 \tag{3.28}$$
$$= e^{2\beta_M\sigma}\left(-dt^2+a^2(t)d\ell^2\right) \tag{3.29}$$
$$= -d\tilde{t}^2+\tilde{a}^2(t)d\ell^2 \tag{3.30}$$

where $d\ell^2=\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)$ is the space part of the space-time interval in a non-expanding universe,

$$d\tilde{t}=e^{\beta_M\sigma}dt \qquad \text{and} \qquad \tilde{a}^2(t)=e^{2\beta_M\sigma}a(t). \tag{3.31}$$

We generalise the previous results to the usual case where $P=\left(\gamma-1\right)\rho$ and we also introduce the new variables

$$\begin{aligned}
\tilde{y} &= \frac{d\sigma}{d\tilde{t}} \\
\tilde{H} &= \frac{1}{\tilde{a}}\frac{d\tilde{a}}{d\tilde{t}} \\
r &= 4-3\gamma
\end{aligned} \tag{3.32}$$

We get the following system of equations

[2] Although this is subject to a heated debated to know whether one, both or neither of those frames are physical is still a practical differential tool.

$$
\begin{aligned}
\frac{dy}{d\tilde{t}} &= -3r\beta_{DM}\tilde{H}^2 + (6r\beta_{DM}\beta_M - 3)\,\tilde{H}\tilde{y} + \left(2\beta_M - 3\beta_{DM}\beta_M^2 + \frac{1}{2}r\beta_{DM}\right)\tilde{y}^2 \\
\frac{dH}{d\tilde{t}} &= \left(-\frac{3}{2}\gamma - 3r\beta_{DM}\beta_M\right)\tilde{h}^2 + \left(3\gamma\beta_M - 4\beta_M + 6r\beta_{DM}\beta_M^2\right)\tilde{H}\tilde{y} \\
&\quad + \left(\frac{1}{4}\gamma - \frac{1}{2} - \frac{3}{2}\gamma\beta_M^1 + 3\beta_M - 3r\beta_{DM}\beta_M^3 + \frac{1}{2}r\beta_{DM}\beta_M\right)\tilde{y}^2
\end{aligned}
\tag{3.33}
$$

A mathematical study of this system has been done by Damour *et al.* [1]. It shows that there is only one attractor in the phase space. Close to this attractor solution of the system, we get

$$
\frac{1}{\tilde{\mathscr{G}}}\frac{d\mathscr{G}}{d\tilde{t}} = -\frac{4\beta_{DM}\beta_M}{\frac{3}{2} + \beta_{DM}^2 - 2\beta_{DM}\beta_M}\frac{1}{\tilde{t}}. \tag{3.34}
$$

The value of $\frac{1}{\tilde{\mathscr{G}}}\frac{d\mathscr{G}}{d\tilde{t}}$ is accessible to measurements *via* the study of solar-system data gathered by the Viking-lander available in 1990 by Hellings *et al.* in [13].
The result of this study shows that

$$
\left|\frac{1}{\tilde{\mathscr{G}}}\frac{d\mathscr{G}}{d\tilde{t}}\right| \leq 2.1 \times 10^{-29} \quad \mathrm{s}^{-1}. \tag{3.35}
$$

The observational data are thus compatible with the existence of an exactly massless dilaton field coupled in such a way with dark matter.
To completely test the model, one has to improve the precision of the $\frac{1}{\tilde{\mathscr{G}}}\frac{d\mathscr{G}}{d\tilde{t}}$ measurements.

Chapter 4

Symmetries for an unnoticed dilaton

THE DILATON FIELD is a plausible alternative to the cosmological constant as we have seen in the previous chapters. According to experiments it has to be nearly massless if not exactly massless. Nevertheless it has not been observed yet and there might be reasons for that. In this chapter we will investigate the symmetries of the dilaton field that could prevent a direct coupling (as opposed to effective coupling through the metric) with Standard Model fields. We will also try to set some experimental bounds on the values of the corresponding possible coupling constants.

Let us consider a massive scalar dilaton field with action

$$\mathcal{S} = \int \left(\mathcal{R}\phi - \omega \frac{\nabla_\mu \phi \nabla^\mu \phi}{\phi} - V(\phi) \right) \sqrt{-g} d^4x + 16\pi \mathcal{S}_M \left[\psi_M, \mathbf{g}_{\mu\nu} \right] + 16\pi \mathcal{S}_{DM} \left[\psi_{DM}, \mathbf{g}_{\mu\nu} \right]$$

with a small mass defined as follows from the potential

$$m_\phi = \sqrt{\frac{V''(\phi)}{2}} \leq H_0 \simeq 10^{-33} \quad \text{eV}.$$

As this field is expected to be very light, the forces it would give rise to would have a very long range (this upper bound corresponds to a correlation length of approximately 1 a.u.).

Coupling to the Standard Model

Let us look at direct couplings of the dilaton field with Standard Model fields of the form

$$\beta_i \frac{\phi}{M} \mathcal{L}_i \tag{4.1}$$

where β_i is a dimensionless coupling constant. As we have no knowledge about it, we expect that β_i is of the order of unity. M is a mass parameter (UV-cutoff) representing the energy scale we integrated out to get our low-energy description. We cannot know its accurate value but it should not be greater than the scale at which quantum gravity is relevant. $\mathcal{L}_i$ is any gauge-invariant 4-operator (*e.g.* $\mathbf{F}^{\mu\nu}\mathbf{F}_{\mu\nu}$ for coupling to Quantum Electrodynamics).

Consequences of a coupling to a dynamical dilaton field

To be relevant today, the dynamics of ϕ implies that it varies at most by the order of M_{Pl} over time scale of the order of H_0^{-1} (the inverse of HUBBLE constant). This can be shown using data from the primordial nucleosynthesis era which would have been very different if ϕ had varied more.
The time evolution of ϕ would also imply variations of other fundamental constants. For example, a coupling such as

$$\beta_{G^2} \frac{\phi}{M} \operatorname{Tr}\left(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}\right),$$

would induce a slight variation of the fine structure constant α. Where $\mathbf{G}$ is the gluon gauge field of Quantum Chromodynamics. These variations can be measured and they give a constraint on the value of the coupling constant.

Experimental constraints on the coupling

With a coupling of the form proposed in equation (4.1) for Quantum Chromodynamics: $\beta_{G^2} \frac{\phi}{M} \operatorname{Tr}\left(\mathbf{G}^{\mu\nu}\mathbf{G}_{\mu\nu}\right)$, it has been shown that

$$|\beta_{G^2}| \leq 10^{-4} \frac{M}{M_{\text{Pl}}}, \tag{4.2}$$

where M_{Pl} is the reduced PLANCK mass. It is possible to get a constraint on other coupling constants. They has be summarised in [14]. All these constraints are not good evidence in favour of the dilaton field. As a matter of fact, these constraints are looser than what is experimentally observable yet. Let us not rule out the existence of the dilaton too soon as there could be explanations for these coupling to be so weak.

Symmetries

An explanation for these very low observed coupling constants could be that the dilaton field obeys symmetries that prevent the coupling to fields of the Standard Model. Let us examine some of such symmetries.

What symmetries are possible for ϕ ?

A symmetry of the form

$$\phi \to \phi + cst \tag{4.3}$$

must be ruled out for it is not compatible with a non zero potential $V(\phi)$. A discrete symmetry, for example of the form

$$\phi \to -\phi \tag{4.4}$$

that could for example arise from a broken gauge symmetry is also to be excluded, as in our case, discrete symmetries are themselves spontaneously broken [2]. The only possibility is then to approximate a symmetry with a symmetry of the form of equation (4.3). Note however that, in the limit of $m_\phi = 0$, this symmetry would be exact.

Quantum gravity

In string theory there are no unbroken global symmetries but in 1998 one did not know enough about quantum gravity to be able to assert if it is always the case. For example string theory has no exact symmetry but within this theory there exists axionlike fields with an approximate Peccei-Quinn symmetry.

It has also been shown that the action depends on the structure of space-time on small scales and that the symmetry suppression could be such that axion is still a solution to the *strong CP problem*[1]. Such an effect is much stronger than the bound established earlier on the coupling with Quantum Chromodynamics.

Although the reliance of an approximate global symmetry in presence of gravity has not been established yet, let us suppose from now on that the dilaton field has not been detected yet because of such a symmetry.

Possible couplings

In this context, let us review the possible approximate couplings of the dilaton field to Standard Model fields.

Simplest form of coupling

We would like to find a simple coupling with a field of the Standard Model that respects the approximate symmetry of the form (4.3). The smallest order scalar operator to satisfy that symmetry is of the form

$$\nabla^\mu \nabla_\mu \phi. \tag{4.5}$$

But this operator is of degree three (as its dimension is length^{-3}) and should as such be divided by M^3, with a coupling as we assumed (4.1), making it negligible. An other way to obtain a coupling abiding by the symmetry constraint is to couple ϕ to a total derivative and make an integration by part, to obtain a $\partial_\mu \phi$ term. The only allowed term in the Standard Model to do so is

$$\beta_{\mathbf{FF}^*} \frac{\phi}{M} \mathbf{F}^{\mu\nu} \mathbf{F}^*_{\mu\nu} = \frac{\beta_{\mathbf{FF}^*}}{M} \left[- \left(\partial_\mu \phi \right) \mathbf{K}^\mu + \partial_\mu \left(\phi \mathbf{K}^\mu \right) \right], \tag{4.6}$$

where $\mathbf{F}$ is the electromagnetic tensor and $\mathbf{F}^*$ its dual, such that $\mathbf{F}^{*\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\mathbf{F}_{\rho\sigma}$. $\mathbf{K}^\mu = 2\mathbf{A}_\nu \mathbf{F}^{*\mu\nu}$ with $\mathbf{A}$ the electromagnetic four-potential. One can check that this coupling respects the constraint we imposed. For latter term it is a divergence and its integral reduces to its value on the border which we send to infinity and make it thus vanish.

Observable consequences

$\mathbf{F}^{\mu\nu}\mathbf{F}^*_{\mu\nu}$ is a pseudo scalar *i.e.* it behaves like a scalar but its sign changes with a parity transformation. As such it can not accumulate coherently in macroscopic test bodies and thus cannot give rise to appreciable long-range forces. However if we make the same assumptions as we made earlier, that is assuming a time-only dependent field, this gives rise to observable effects. The polarisation of light from distant radio sources would be rotated by the field as the dispersion relation for electromagnetic radiation becomes

$$\omega^2 = k^2 \pm \frac{\beta_{\mathbf{FF}^*}}{M} \dot{\phi} k, \tag{4.7}$$

[1] *i.e.* the fact that Quantum Chromodynamics does not respect the Charge-Parity symmetry.

where the + sign is for right-handed circularly polarised light and the − sign is for left-handed circularly polarised light. The overdot is for time derivation.
If we define $\Delta\chi$ the angular change of polarisation, it can be measured observing distant radio galaxies and quasars. These objects have a known relationship between their luminosity structure and polarisation structure. With the above dispersion relation we get the following expression in the JWBKLG[2] limit where the electromagnetic wavelength is much smaller than the one of ϕ. The difference of group velocity for the right-handed and left-handed modes leads to a polarisation rotation of

$$\Delta\chi = \frac{\beta_{\mathbf{FF^*}}}{M}\left(\phi(t_{\text{emitted}}) - \phi(t_{\text{now}})\right). \tag{4.8}$$

Let us write $\Delta\phi = \phi(t_{\text{emitted}}) - \phi(t_{\text{now}})$ from now on. Data from observation [15] of such objects has been analysed and enables obtaining a constraint on this coupling.

$$|\beta_{\mathbf{FF^*}}| \leq 3 \times 10^{-2} \times \frac{M}{|\Delta\phi|}. \tag{4.9}$$

In summary the experimentally obtained constraint were at the end of the nineties too loose to conclude on the existence of the dilaton field. As a matter of fact no signature of a dilaton field has been detected with the experimental bound. And no symmetry seems strong enough so as to prevent interaction with Standard Model fields that would enable detection. In addition to that, some of the above discussed arguments come from string theory that had been trending for long at that time. Now some other quantum gravity theories have gained popularity and could provide with new arguments to explain the non-observation of the dilaton field.

[2] Jeffreys-WentzelKramersBrillouin-Liouville-Green

Chapter 5

Consequences on the Microwave Background

THE COSMOLOGICAL MICROWAVE BACKGROUND is a very rich source of data on the primordial universe. Let us study the consequences of a dilation field coupled explicitly to matter as in chapter 2 on the Microwave Background.

The modified model

As we defined energy-momentum tensors for matter and dark matter in chapter two, one can define an energy-momentum tensor for the ϕ field. Let us call it $\mathbf{T}_\phi$ and let us write $\mathbf{T} = \mathbf{T}_{DM} + \mathbf{T}_M$. Conservation law imply that

$$\nabla^\mu \left(\mathbf{T}_{\mu\nu} + \mathbf{T}_{\phi\mu\nu}\right) = 0 \tag{5.1}$$

as, by definition it is the NŒTHER current associated with translation invariance. Equation (3.5) thus implies that, with α a constant,

$$\begin{aligned} \nabla^\mu \mathbf{T}_{\mu\nu} &= -\alpha \operatorname{Tr}(\mathbf{T}) \nabla_\nu \phi \\ \nabla^\mu \mathbf{T}_{\phi\mu\nu} &= \alpha \operatorname{Tr}(\mathbf{T}) \nabla_\nu \phi \end{aligned} \tag{5.2}$$

Here we understand that the model is somewhat simplified as the coupling is supposed to be the same for regular matter and dark matter. It is worth noting that the coupling to radiation vanishes as $\operatorname{Tr}\mathbf{T}_{\mathrm{rad}} = 0$.
We can have different bounds, which we discussed earlier, on the coupling constant α but they would be local in space-time and could be not respected. We also have the constraint coming from the era of primordial nucleosynthesis: the contribution of the field has to be small enough so that the production of elements is not disturbed. This implies

$$\Omega_\phi(t_{\mathrm{ns}}) \lesssim 0.2. \tag{5.3}$$

The previous notation comes from the usual re-writing of FRIEDMANN equation that follows

$$H^2 = H_0^2 \left(\frac{\Omega_k}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_{\mathrm{rad}}}{a^4} + \Omega_\phi\right), \tag{5.4}$$

where each Ω (depending on time) stands for the contribution of the curvature, matter, radiation and the dilaton field respectively. We will take the conformal version of the FRIEDMANN-LEMAÎTRE-ROBERTSON-WALKER metric which reads

$$ds^2 = a^2(t)\left(-d\tau^2 + \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right) \tag{5.5}$$

and which we will use in the flat case ($k = 0$). We can rewrite equation (3.27) with a massive field

$$\ddot{\phi} + 2H\dot{\phi} + a^2\frac{\partial V}{\partial \phi} = \alpha\rho_M a^2 \tag{5.6}$$

and we take an exponential potential of the following form in equation (2.8)

$$V(\phi) = Ae^{\sqrt{2/3}\kappa\mu\phi}. \tag{5.7}$$

We thus get the following matter and radiation equations

$$\begin{aligned} \dot{\rho}_M + 3H\rho_M &= -C\rho_M\dot{\phi} \\ \dot{\rho}_{\mathrm{rad}} + 4H\rho_{\mathrm{rad}} &= 0 \end{aligned} \tag{5.8}$$

One can rewrite the equation in the following simpler form

$$\begin{aligned}
x' &= x\left(\frac{z'}{z}-1\right) - \mu y^2\beta\left(1-x^2-y^2-z^2\right) \\
y' &= \mu x y + y\left(2+\frac{z'}{z}\right) \\
z' &= -\frac{z}{2}\left(1-3x^2+3y^2-z^2\right)
\end{aligned} \tag{5.9}$$

with

$$\begin{aligned}
x &= \frac{\kappa}{H}\frac{\dot{\phi}}{\sqrt{6}} \\
y &= \frac{\kappa a}{H}\sqrt{\frac{U}{3}} \\
z &= \frac{\kappa a}{H}\sqrt{\frac{\rho_{\text{rad}}}{3}}
\end{aligned} \tag{5.10}$$

Solutions

The study of the phase space proposed in [3, 4] shows that there are only two types of solution that show an acceleration of the expansion of the universe. This expansion being observed we restrain the study to these two kinds of solution. However only one of them has a *matter dominated era*. As it has been experimentally shown that this era happened, let us examine the only type of solution that is consistent with experiments.

The scenario described by this solution has a matter density that goes to zero with time *i.e.* an infinite expansion of the universe. It is also known that the actual value of Ω_ϕ is 0.7. Thus we can choose initial condition that will give us such a value and then compare the resulting Cosmological Microwave Background to the one we observe.

Comparison to the Cosmological Microwave Background

If one chooses appropriate final values for the other Ω parameters that are consistent with what can be observed today, a bound for β can be obtained comparing the prediction of the multipole spectrum of the cosmological microwave background.

$$|\beta| < 0.1 \tag{5.11}$$

The ϕ-matter dominated era is when the dilaton field and matter are of equal importance and it is a particular point of the phase space of the dynamical system. We may have thought that during the ϕ-matter dominated era the phase space behaviour of the dynamical system could be used to constrain the value of μ. But the predicted behaviour does not depend on μ. So it is not possible to obtain a constrain on its value.
This thus shows that to agree with experiments the coupling constant of the dilaton field with matter and dark matter (which we assumed to be the same in this chapter) has to be bounded. This still does not enable to conclude whether this field exists, but more recent data coming from the analysis of the Cosmological Microwave Background may give us a sharper bound or maybe prove the existence of this field.

Chapter 6

Conclusion

THE DILATON FIELD has been of great interest as a solution to the Dark Energy problem at the end of the twentieth century. Even though it has not been observed yet, it is interesting to go through its construction.

It was first built on the idea of MACH that inertia comes from distant mass and on the weak equivalence principle, to evaluate the possibility of a time-varying NEWTON's gravitational "constant" [6]. The coupling of the dilaton field to matter and dark matter has then be studied to get an expression of the variation of the gravitational constant due to the existence of such a field [1]. This value can then be compared to experimental data in order to get a bound on the value on the coupling constants.

In [2] it was then tried to understand why the dilaton field has not been detected yet, by studying the possible symmetries of the dilaton field that could limit its observability. This has allowed testing different sorts of simple couplings with the fields of the Standard Model, that had been neglected in the first place. This study has enabled putting new bounds on the coupling constants of the dilaton fields with other fundamental degrees of freedom.

Finally [3, 4] considered the consequences of the existence of the dilaton field on the Cosmological Microwave Background, as it is a very rich source of data regarding the past of the universe. Choosing appropriate values model parameters to model the multipolar spectrum of the Cosmological Microwave Background has led to stronger bounds on the coupling constant of the dilaton field with matter and dark matter.

The dilaton field is still today being used as part of alternative cosmological models (*e.g.* holography theory, AdS/CFT correspondence...). The main cosmological models are nowadays the ones that use a cosmological constant which are refered to as ΛCDM. These ones do not assume a dynamical field for Dark Energy. A quick scan of the literature has not shown many paper invoking the ϕCDM model since the ones I studied in this report.

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